Electroweak Physics Reloaded

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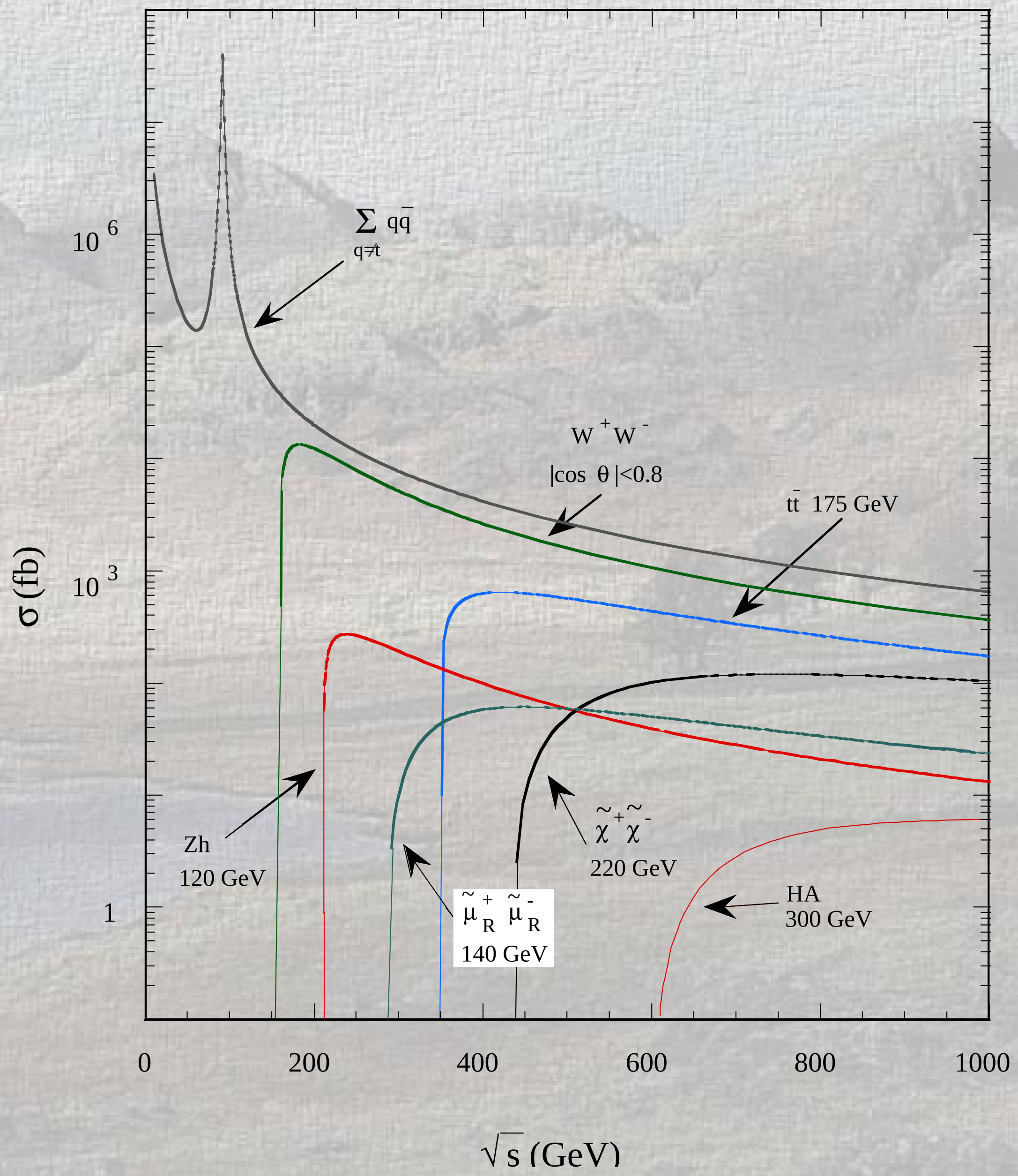
Production cross sections at a Linear Collider

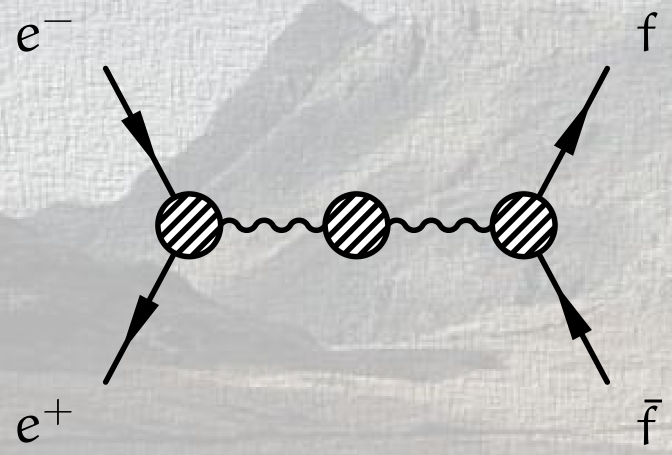
- dominated by electroweak processes
  - $e^+e^- \to 2f$
  - $e^+e^- \to W^+W^- \to 4f$

☹ even if electroweak physics is not perceived as as glamorous as the glimmer twins SUSY and Higgs . . .

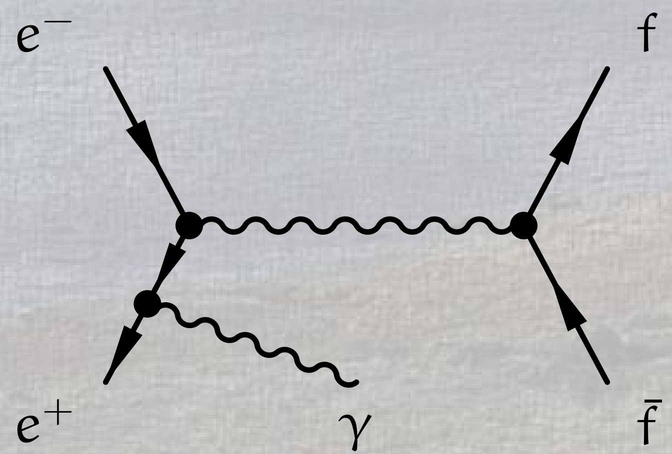
☺ . . . there will be a lot of information, allowing precision measurements and tests, just waiting to be harvested.



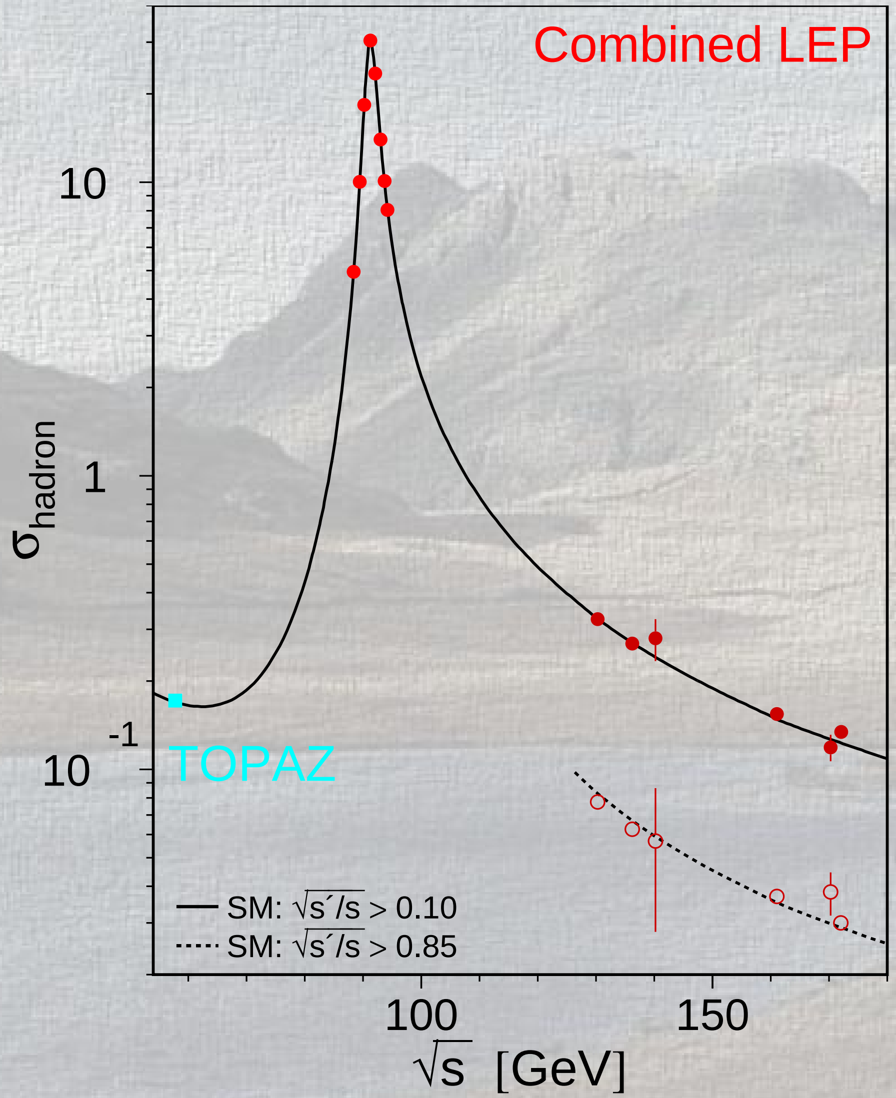
- Mission: make as many Zs as possible!



☺ precision measurements of $Zf\bar{f}$ couplings and Z properties



$\therefore$ initial state ratiation: $s'_{f\bar{f}} \leq s_{e^+e^-}$


Combined LEP
σ_{hadron}
10
1
10^{-1}
TOPAZ
SM: $\sqrt{s'/s} > 0.10$
SM: $\sqrt{s'/s} > 0.85$
100
150
$\sqrt{s}$ [GeV]

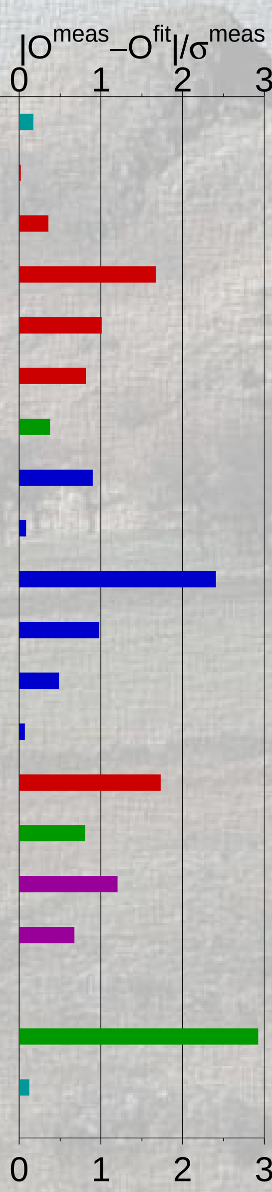
(Winter 1997)

Combined fit of 20 observables

- mass and width of the Z and W
- mass of the top quark
- hadronic cross section at the peak
- 3 BRs $R_{\{\ell,c,b\}}$
- 3 forw./backw. asymm. $A_{FB}^{0,\{\ell,c,b\}}$
- 3 $A_{\{\ell,c,b\}}$
- τ polarization asymmetry
- $\sin^2\theta_{\text{eff.}}^{\text{lept.}}$ from charge asymmetry
- $\sin^2\theta_W$ in $\nu N \to e^- + X$ DIS
- running of the QED coupling
- atomic parity violation

Summer 2003

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02761 ± 0.00036	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1875
Γ_Z [GeV]	2.4952 ± 0.0023	2.4960
σ_{had}^0 [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01636
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1477
R_b	0.21638 ± 0.00066	0.21579
R_c	0.1720 ± 0.0030	0.1723
$A_{fb}^{0,b}$	0.0997 ± 0.0016	0.1036
$A_{fb}^{0,c}$	0.0706 ± 0.0035	0.0740
A_b	0.925 ± 0.020	0.935
A_c	0.670 ± 0.026	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1477
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.426 ± 0.034	80.385
Γ_W [GeV]	2.139 ± 0.069	2.093
m_t [GeV]	174.3 ± 5.1	174.3
$\sin^2\theta_W(\nu N)$	0.2277 ± 0.0016	0.2229
Q_W(Cs)	-72.84 ± 0.46	-72.90



NB: Fitting 20 observables is a non trivial test, because most of the 19 parameters of the standard model come from the light flavor sector and do not enter the observables significantly.

Nowever, there are some “black sheep”:

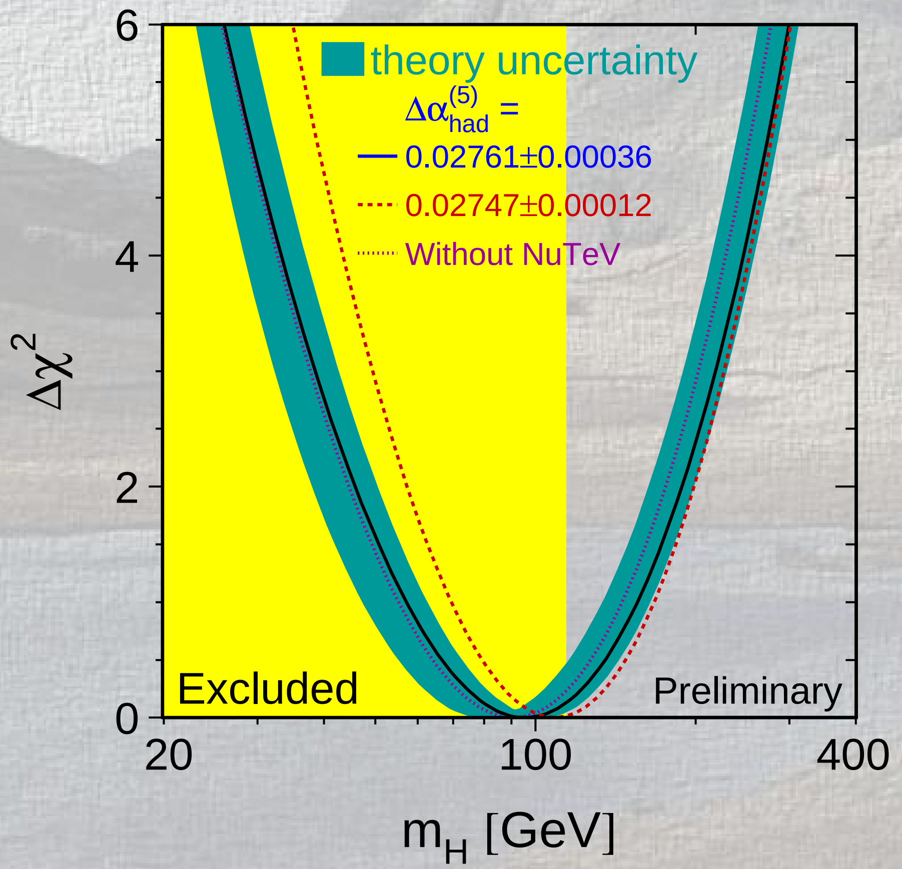
- The NuTeV measurement (final) of $\sin^2\theta_W$ in $\nu N \to e^- + X$ DIS is off by almost 3σ
- $A_{FB}^{0,b}$ is off by more than 2σ
- the hadronic cross section at the peak is off by less than 2σ
- The SLD measurement A_ℓ is off by less than 2σ

The NuTeV-result could be a cause for concern, but the probability for 2σ-fluctuations is around 5%.

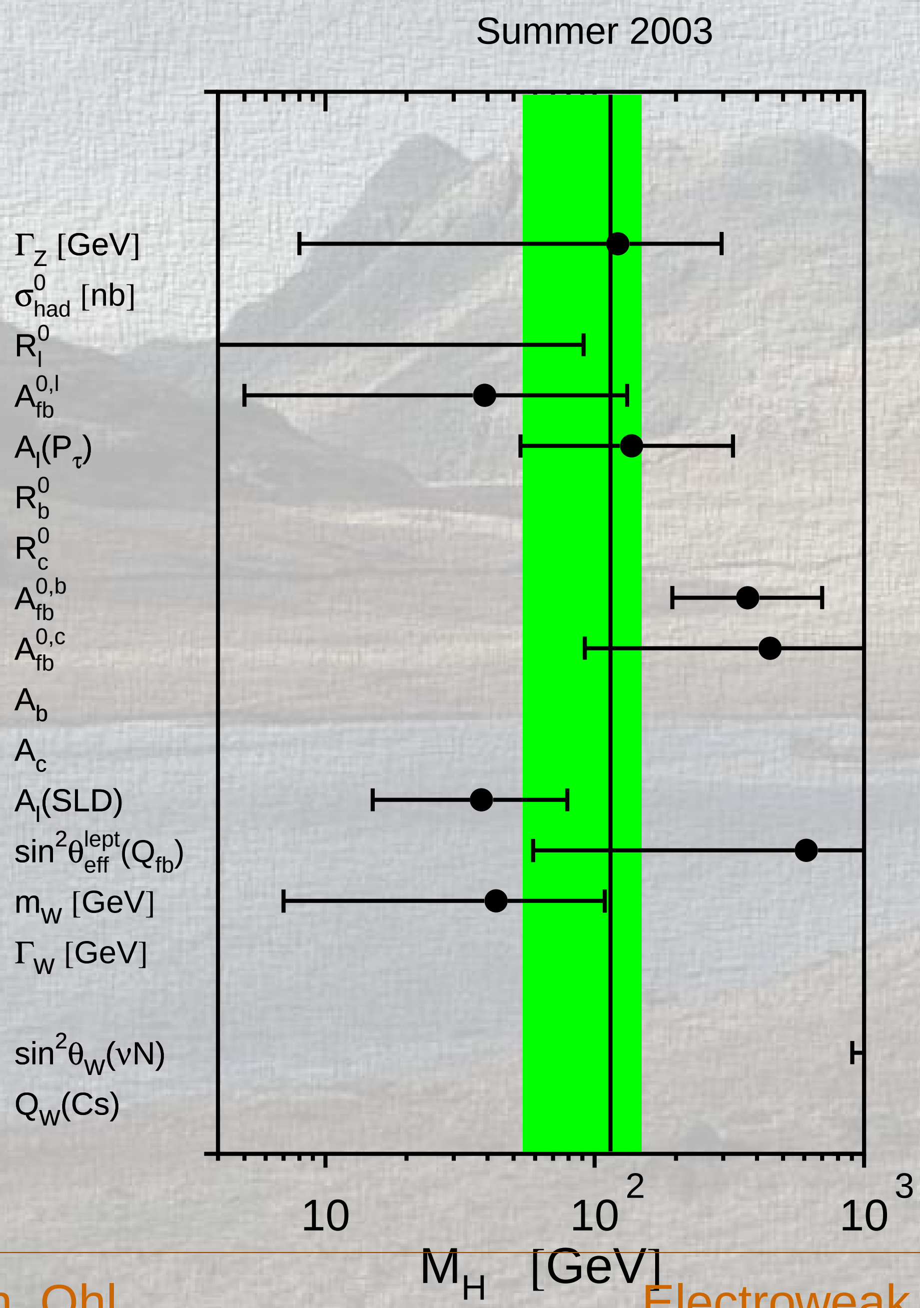
- below, we will see that the NuTeV result is also inconsistent with the m_W measurements at LEP2, Tevatron (in addition to the LEP2/SLC(/m_t) results)

The Blue Band Plot gives the quality of fit as function of the (SM-)Higgs mass

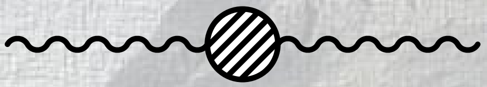
frozen at their measured values:



Repeat fits with individual parameters


Summer 2003
Γ_Z [GeV]
σ^0_{had} [nb]
R^0_l
$A^{0,l}_{fb}$
$A_l(P_\tau)$
R^0_b
R^0_c
$A^{0,b}_{fb}$
$A^{0,c}_{fb}$
A_b
A_c
A_l(SLD)
$\sin^2\theta^{lept}_{eff}(Q_{fb})$
m_W [GeV]
Γ_W [GeV]
$\sin^2\theta_W(\nu N)$
Q_W(Cs)
10
10^2
10^3
M_H [GeV]

Gauge boson self energies are an important class of corrections. The Standard Model prediction must be subtracted before making claims of new physics

V  V' $= i\pi_{VV'}(Q^2) = i\pi^{\text{SM}}_{VV'}(Q^2) + i\pi^{\text{BSM}}_{VV'}(Q^2)$

Popular parametrization by three parameters S, T and U [Peskin/Takeuichi]

- Breaking of custodial $SU(2)$:

$$\alpha(m_Z)\cdot T = \frac{\pi^{\text{BSM}}_{WW}(0)}{m_W^2} - \frac{\pi^{\text{BSM}}_{ZZ}(0)}{m_Z^2}$$

- non standard running of m_Z ($[\ldots]$ can be omitted in $\overline{\text{MS}}$ renormalization):

$$\frac{\alpha(m_Z)}{4\cos^2\theta_w(m_Z^2)\sin^2\theta_w(m_Z^2)}\cdot S = \frac{\pi^{\text{BSM}}_{ZZ}(m_Z^2) - \pi^{\text{BSM}}_{ZZ}(0)}{m_Z^2} - \left[\frac{\cos^2\theta_w(m_Z^2) - \sin^2\theta_w(m_Z^2)}{\cos\theta_w(m_Z^2)\sin\theta_w(m_Z^2)}\frac{\pi^{\text{BSM}}_{Z\gamma}(m_Z^2)}{m_Z^2} + \frac{\pi^{\text{BSM}}_{\gamma\gamma}(m_Z^2)}{m_Z^2}\right]$$

- non standard running of m_W ($[\ldots]$ can be omitted in $\overline{\text{MS}}$ renormalization):

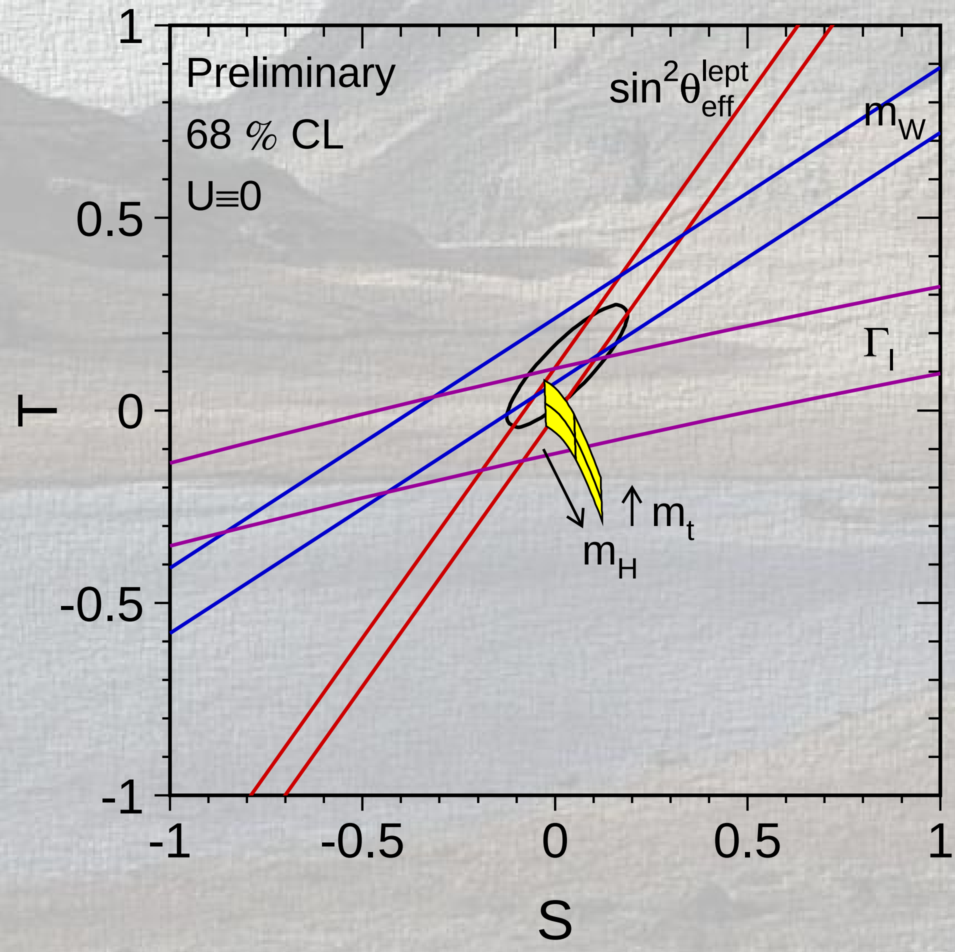
$$\frac{\alpha(m_Z)}{4\sin^2\theta_w(m_Z^2)}\cdot(S+U) = \frac{\pi_{WW}^{\text{BSM}}(m_W^2)-\pi_{WW}^{\text{BSM}}(0)}{m_W^2} - \left[\frac{\cos\theta_w(m_Z^2)}{\sin\theta_w(m_Z^2)}\frac{\pi_{Z\gamma}^{\text{BSM}}(m_Z^2)}{m_Z^2}+\frac{\pi_{\gamma\gamma}^{\text{BSM}}(m_Z^2)}{m_Z^2}\right]$$

Procedure:

1. calculate $\pi_{VV'}^{\text{SM}}(Q^2)$ as precisely as possible
2. measure $\pi_{VV'}(Q^2)$ as precisely as possible
3. infer $\pi_{VV'}^{\text{BSM}}(Q^2) = \pi_{VV'}(Q^2) - \pi_{VV'}^{\text{SM}}(Q^2) \neq 0$
4. find a new model that produces exactly the required $\pi_{VV'}^{\text{BSM}}(Q^2)$
5. either find new physics, buy a frock and book a flight to Stockholm, or

repeat from 1, 2, 3, or 4 ad infinitum

$U = 0$:



The parameters are normalized such that one can expect $|S|, |T|, |U| = \mathcal{O}(1)$ for “typical” models of new physics

- the measurements prefer $S = T = U = 0$
- the change in the SM loop contributions from increasing the Higgs mass would have to be balanced by new physics with $S > 0$ and $T < 0$
- the sensitivity to m_H is weak since it enters the SM loop corrections only logarithmically

- there's no way that Giga-Z could improve m_Z: the beam energy can be calibrated much better in storage rings ("resonant depolarization")
- SLC had polarization, but little luminosity, while LEP had luminosity galore and no polarization
- Giga-Z can make a big impact with e^- and e^+ polarization and high luminosity
  - e^+ polarization loosens the demands on both e^+ and e^- polarimetry by allowing to use a Blondel scheme. For

$$\sigma = [1 - P_{e^+}P_{e^-} + (P_{e^+} - P_{e^-})A_{LR}]\,\sigma_{\text{unpol.}}$$

we have

$$A_{LR} = \sqrt{\frac{(+\sigma_{++} + \sigma_{-+} - \sigma_{+-} - \sigma_{--})(-\sigma_{++} + \sigma_{-+} - \sigma_{+-} + \sigma_{--})}{(+\sigma_{++} + \sigma_{-+} + \sigma_{+-} + \sigma_{--})(-\sigma_{++} + \sigma_{-+} + \sigma_{+-} - \sigma_{--})}}$$

where most of the uncertainties drop out.

Giga-Z (together with improved m_W and m_t from higher energy linear collider experiments) can improve the precision dramatically.

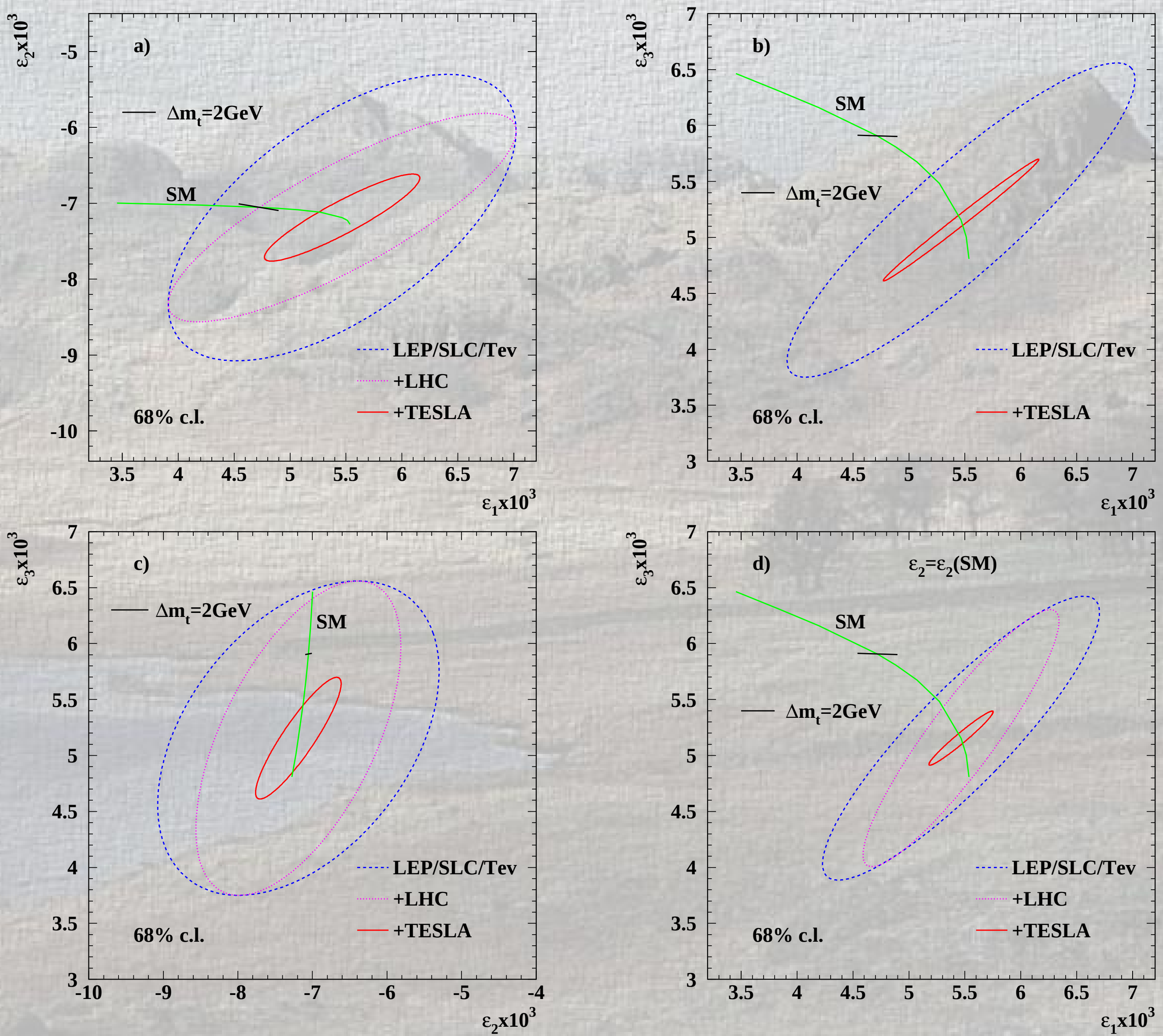
NB:

$$\epsilon_1 = \alpha T$$

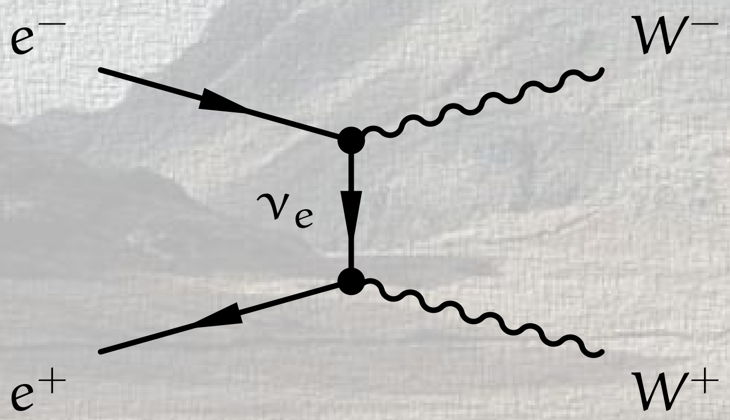
$$\epsilon_2 = -\frac{\alpha}{4\sin^2\theta_w} U$$

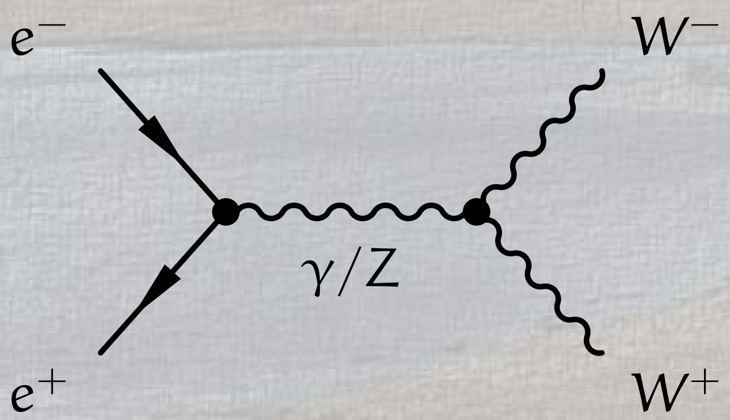
$$\epsilon_3 = \frac{\alpha}{4\sin^2\theta_w} S$$

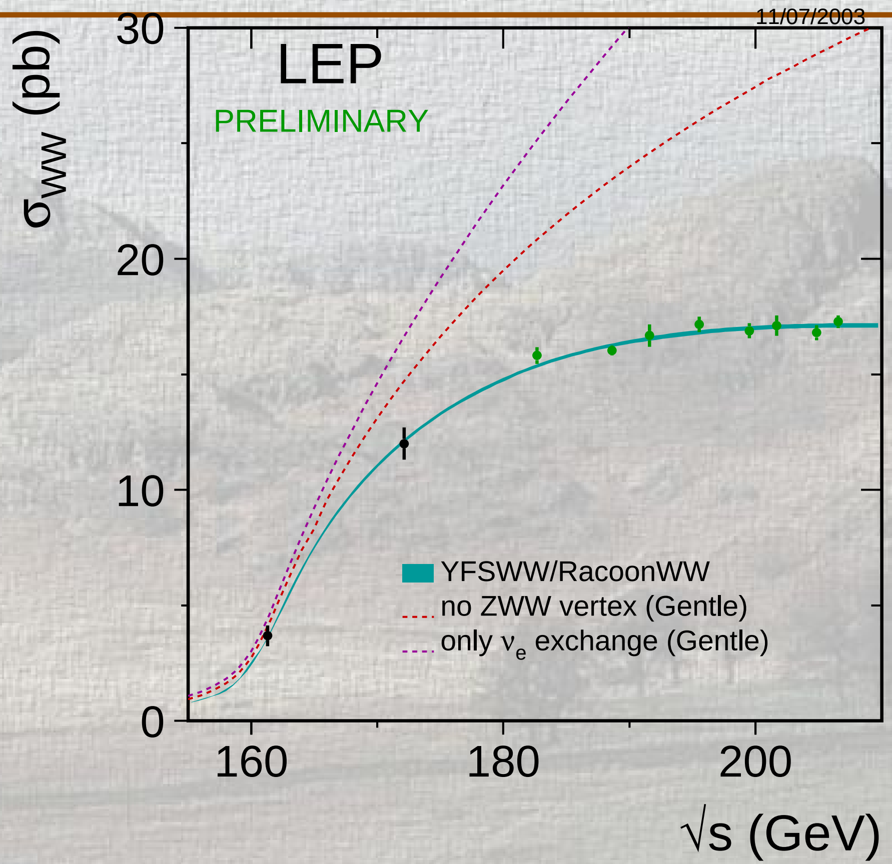
The green SM line corresponds to varying m_H from 70 GeV to 1 TeV.



- Look for the Higgs and, while you're at it, make as many W^+W^- pairs as possible

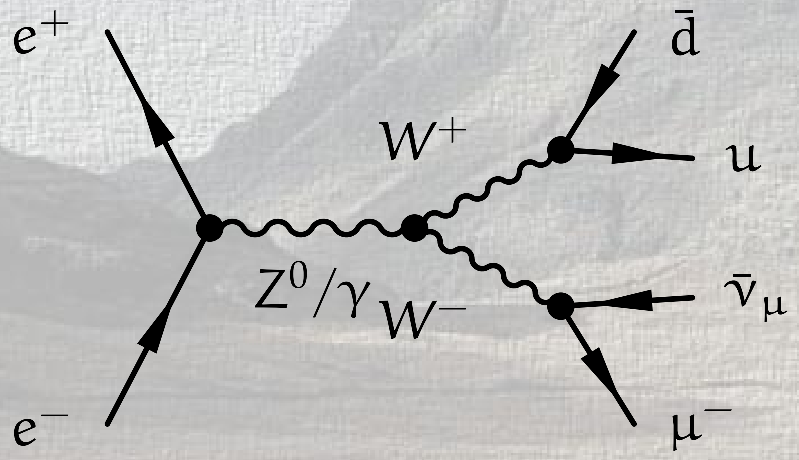






both the $W^+W^-\gamma$ (QED) and the W^+W^-Z (non abelian) couplings have been confirmed impressively

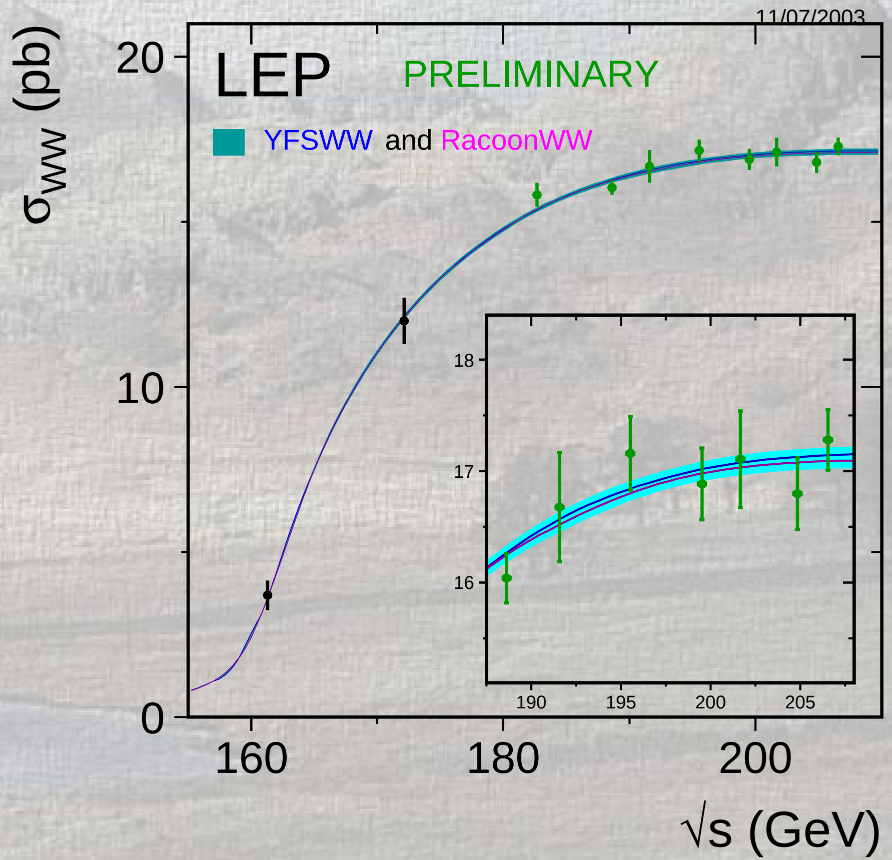
The Ws produced at LEP2 are not stable and decay into fermion pairs



$+9$ additional diagrams

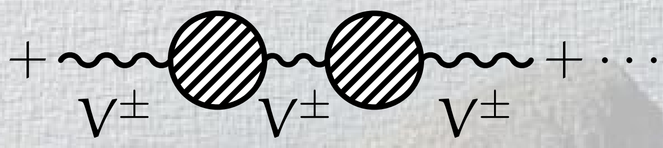
- ☹ A full one loop calculation has never been completed
- ☺ a “good enough for LEP2” calculation in a consistent approximation (“Double Pole Approximation”, DPA) exists [Denner, Dittmaier et al.]

and agrees very well with the experimental result



W mass measurement

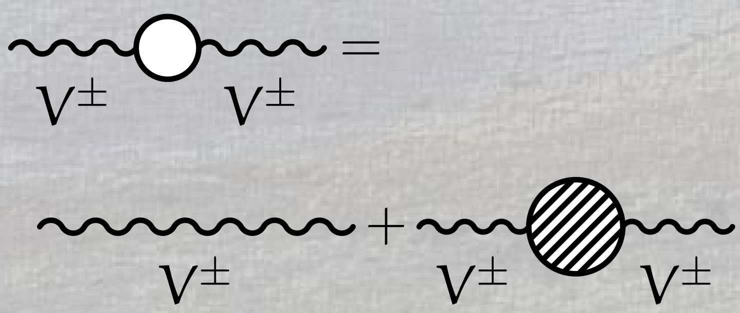
- threshold scan of the total cross section
- invariant mass of decay products

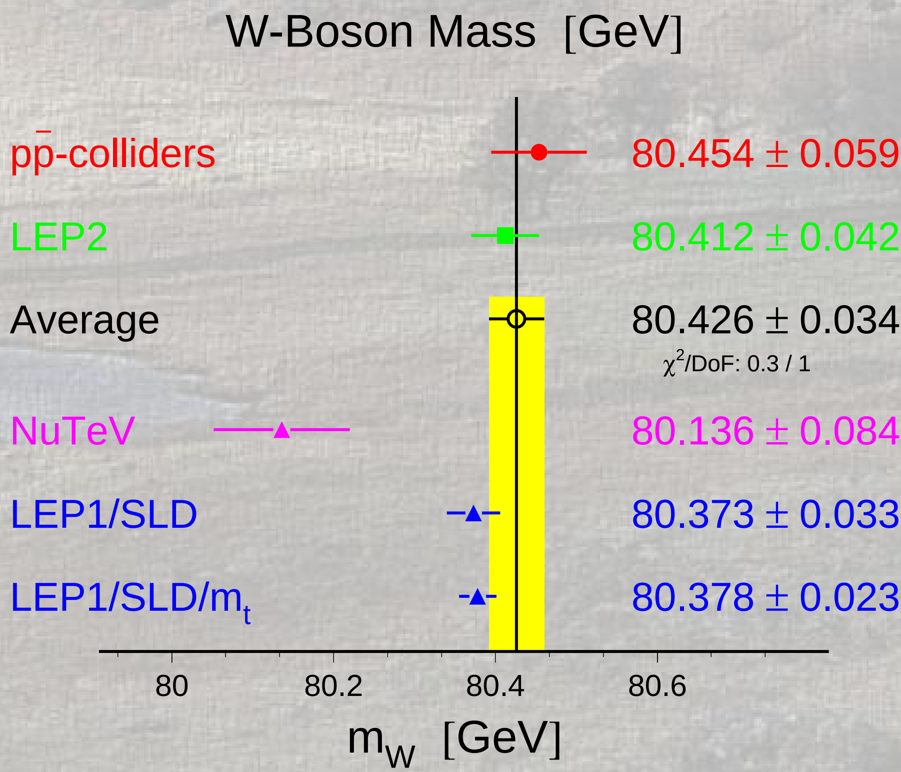


is impossible because hard photons couple to the $W^{\pm}$.

Problems

- ☹ a gauge invariant separation of initial and final state radiation is impossible.
- ☹ a naive Dyson resummation of the *W* propagator





Parametrize the most general W^+W^-V-vertex (where $V = Z, \gamma$) with $U(1)$ gauge invariance:

$$
\begin{aligned}
\mathcal{L}_{WWV}/(-g_{WWV}) = &\\
& i g_1^V \left(W^\dagger_{\mu\nu} W^\mu - W^\dagger_\mu W^\mu{}_\nu \right) V^\nu + i \kappa_V W^\dagger_\mu W_\nu V^{\mu\nu} + i \frac{\lambda_V}{m_W^2} W^\dagger_{\lambda\mu} W^\mu{}_\nu V^{\nu\lambda} \\
& - g_4^V W^\dagger_\mu W_\nu \left(\partial^\mu V^\nu + \partial^\nu V^\mu \right) + g_5^V \epsilon^{\mu\nu\lambda\sigma} \left(W^\dagger_\mu \overleftrightarrow{\partial}_\lambda W_\nu \right) V_\sigma \\
& + i \tilde{\kappa}_V W^\dagger_\mu W_\nu \tilde{V}^{\mu\nu} + i \frac{\tilde{\lambda}_V}{m_W^2} W^\dagger_{\lambda\mu} W^\mu{}_\nu \tilde{V}^{\nu\lambda} \qquad (1)
\end{aligned}
$$

Notation: V^μ stands for either the photon or the Z field, W^μ is the W^- field, $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$, $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, and $\tilde{V}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\sigma} V^{\lambda\sigma}$.

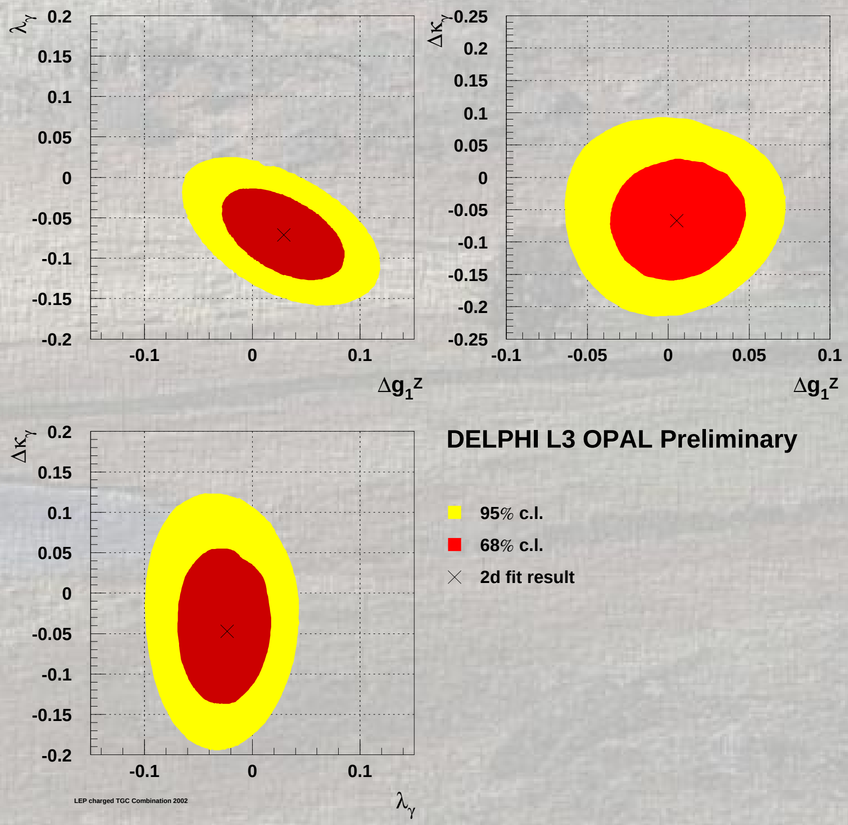
- to cover the most general case, all couplings must be allowed to depend on the momenta entering the vertices

this corresponds to an infinite number of free parameters ...

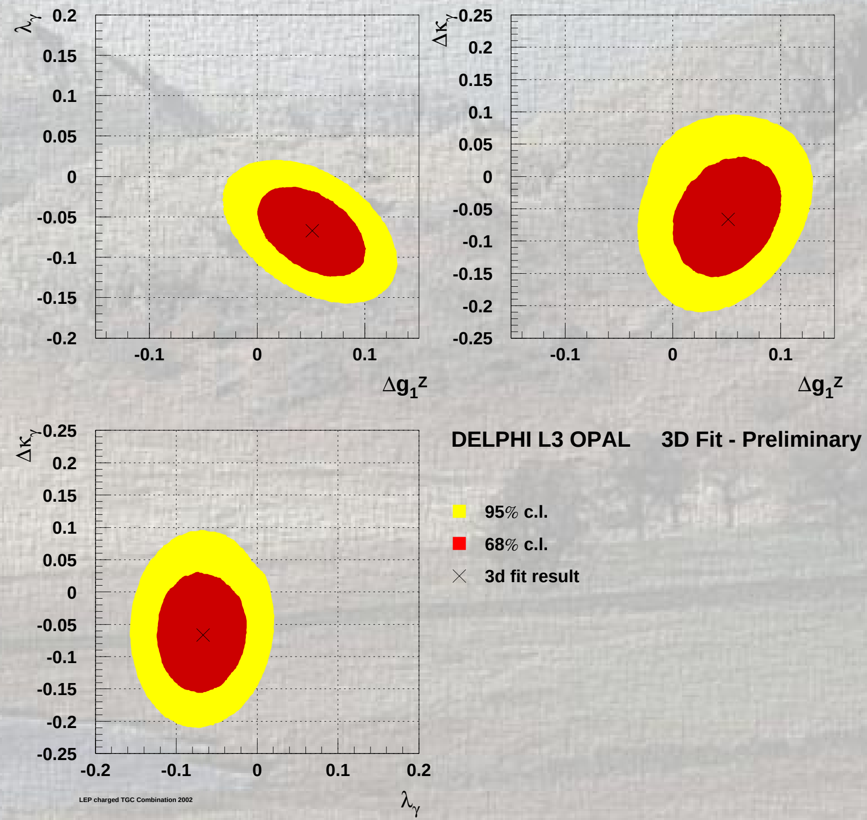
- Starting point: analyze the case of constant κ_γ, λ_γ and g_1^Z.

☺ they appear to be most accessible at LEP2 and do conserve C and P.

two-parameter fits



three-parameter fits

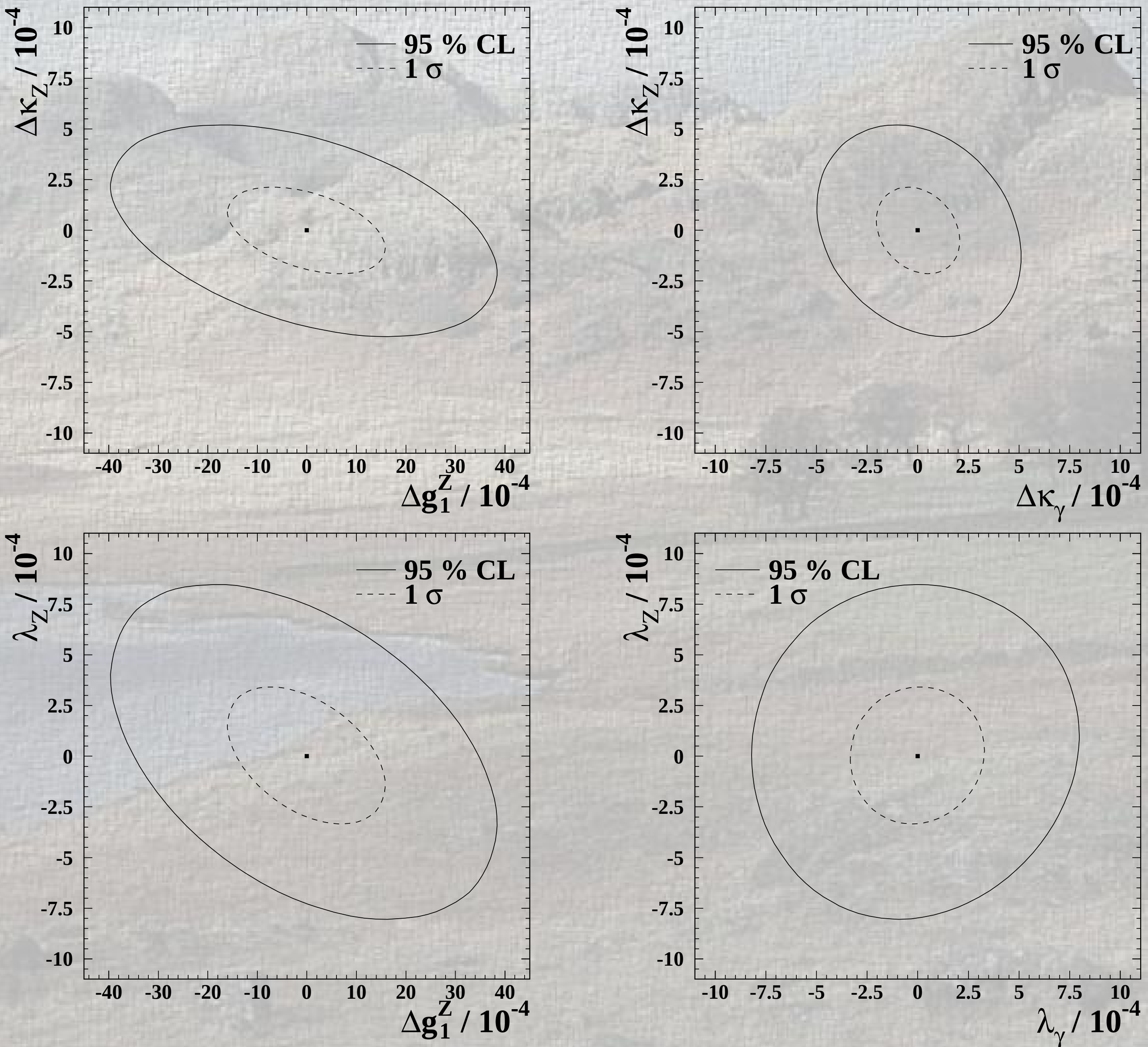


one-parameter fits for TESLA (constant $\kappa_{\gamma,Z}$, $\lambda_{\gamma,Z}$ and g_1^Z at $800\,\mathrm{GeV}$, $1000\,\mathrm{fb}^{-1}$, $P_{e^-} = 80\%$, and $P_{e^+} = 60\%$)

$g_1^Z - 1$	$12.6 \cdot 10^{-4}$
$\kappa_\gamma - 1$	$1.9 \cdot 10^{-4}$
λ_γ	$3.3 \cdot 10^{-4}$
$\kappa_Z - 1$	$1.9 \cdot 10^{-4}$
λ_Z	$3.0 \cdot 10^{-4}$

☺ that's two orders of magnitude better!

five-parameter fit



By now, you will have spotted a trend;

☺ higher energy $\rightarrow$ more observed particles

- not only more particles, but also more particles with individually tagged flavor

$\because$ LHC can do high multiplicities, but most of it are QCD showers

most observed particles are fermions (bound states like pions and nucleons don't count . . .)

- charged leptons (τs play a special rôle)
- hard photons
- b-quarks (probably also c-quarks)
- missing energy and momentum

☺ couplings of leptons and quarks to the known gauge bosons γ, Z, $W^{\pm}$ and g are very well known (from LEP1, LEP2, B-factories and earlier experiments)

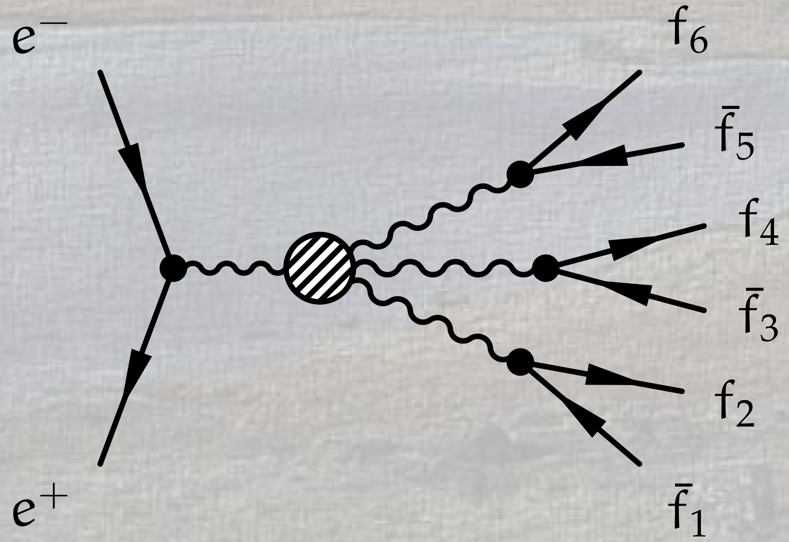
$\therefore$ use leptons and quarks as analyzers for gauge bosons

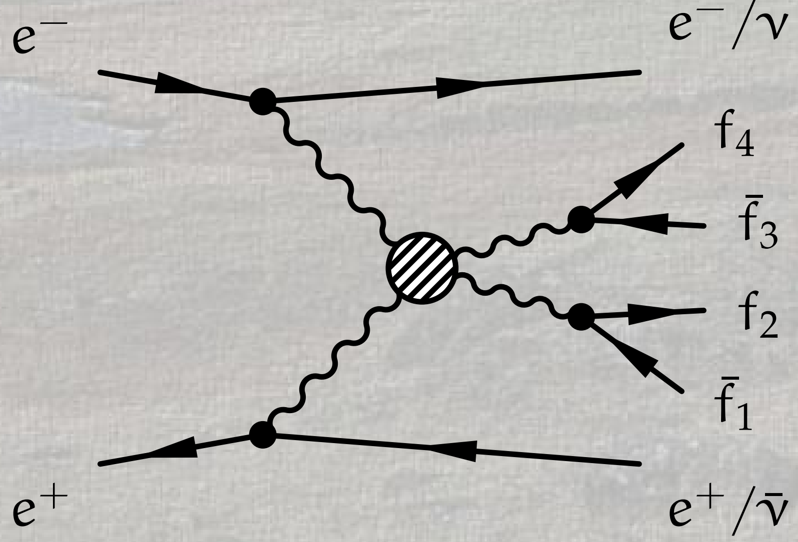
Rules of thumb:

- n gauge boson production $\to 2n$ fermion production
- couplings of n gauge boson can be measured in $(n-1)$ gauge boson production, i. e. $2(n-1)$ fermion production

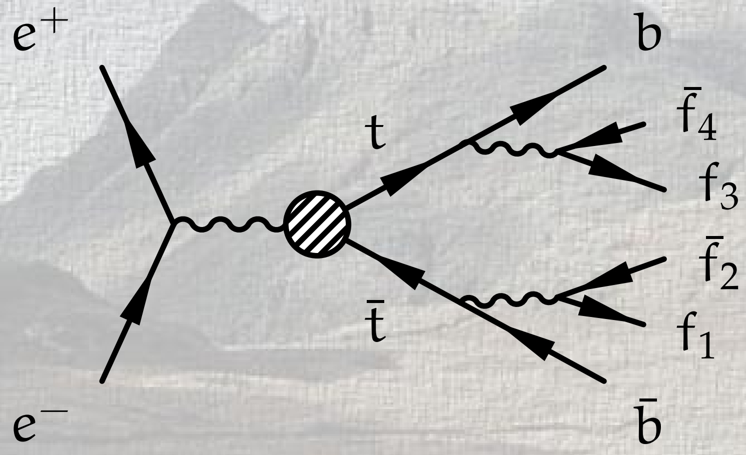
Z mass and width	$2(2-1) = 2$ fermions	LEP1/SLC
triple gauge couplings	$2(3-1) = 4$ fermions	LEP2
quartic gauge couplings	$2(4-1) = 6$ fermions	Linear Collider

☺ these rules also work for gauge boson fusion processes



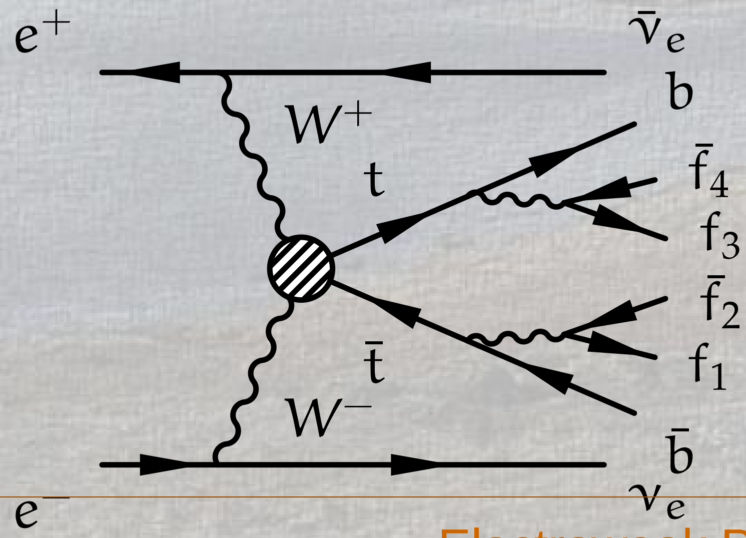


- top quark pair production (single top is only possible at hadron colliders) corresponds to 6 fermion production (i. e. $t\bar{t} \to W^+ b W^- \bar{b} \to f_1 \bar{f}_2 b f_3 \bar{f}_4 \bar{b}$)



  - $e^+e^- \to b u \bar{d} \bar{b} \mu^- \bar{\nu}_\mu$: 232 diagrams on tree level
  - $e^+e^- \to b u \bar{d} \bar{b} e^- \bar{\nu}_e$: 464 diagrams on tree level

- W^+W^- fusion into top quark pairs corresponds to 8 fermion production



  - $e^+e^- \to \nu_e \bar{\nu}_e b u \bar{d} \bar{b} \mu^- \bar{\nu}_\mu$: 6676 diagrams on tree level
  - $e^+e^- \to \nu_e \bar{\nu}_e b u \bar{d} \bar{b} e^- \bar{\mu}_e$: 21058 diagrams on tree level

- ☺ Polarization can be controlled for initial state fermions (certainly for e^-, probably for e^+ — even transversal polarization)
- polarization can not be measured with the usual detectors for final state quarks and leptons (except τs)
- ☺ polarization of intermediate gauge bosons can be inferred from the decay angle distributions (not event by event, though)

$$\frac{d\Gamma}{d\cos\theta_*}(W^-_{S=\pm1} \to \ell\bar{\nu}_\ell) \propto (1 \pm \cos\theta_*)^2$$
$$\frac{d\Gamma}{d\cos\theta_*}(W^-_{S=0} \to \ell\bar{\nu}_\ell) \propto \sin^2\theta_*$$

∴ calculation of gauge boson production and independent decay not sufficient

- Two approaches
  1. do a full calculation of all $2 \to 2n$ Feynman diagrams
  2. calculate production and decay including complete spin correlations.

Both approaches are feasible

- complete tree level calculations of $2 \to 8$ (or even $2 \to 10$) processes are possible today with the aid of automated simulation systems:

  lagrangian, inital and final state $\}$ $\to$ matrix element

  inital and final state, parameters, cuts $\}$ $\to$ phase space

  matrix element, phase space $\}$ $\to$ event sample

- nobody has a working system today, that extends the automated systems to loop level
- there a lot of ideas floating around and the Japanese GRACE group has a system for $2 \to 2$ that appears to be reliable without expert intervention
- though not perfect, the automated systems are almost always good enough at tree level, but hand crafted solutions are required at loop level

- spin correlations can be taken into account easily using a density matrix formalism

initial state → final state

$$\rho_{e^+} \otimes \rho_{e^-} \to T\,(\rho_{e^+} \otimes \rho_{e^-})\,T^\dagger$$

where the scattering matrix T can be decomposed approximately, e. g.

$$T_{e^+e^-\to 4f} \approx (T_{W^+\to 2f} \otimes T_{W^-\to 2f})\,T_{e^+e^-\to W^+W^-}$$

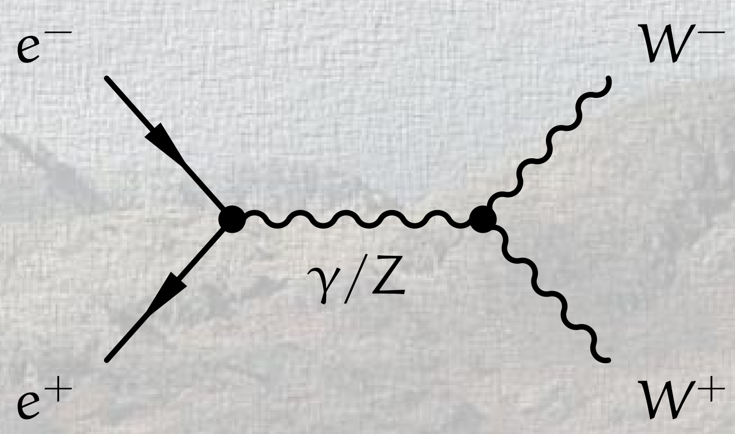
this decomposition is only well defined, if the intermediate particles are strictly on shell, i. e. $(p_{f_1} + p_{\bar{f}_2})^2 = m_W^2$!

off shell, i. e. $(p_{f_1} + p_{\bar{f}_2})^2 \neq m_W^2$, gauge invariance is violated!

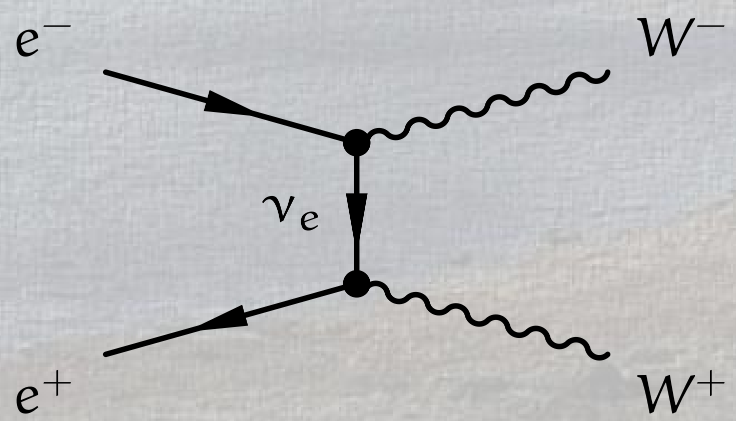
- interacting vector bosons can only be formulated as gauge theories (spontaneously broken, if necessary) to allow to get rid of the superfluous degrees of freedom (4 vector indices vs. two or three polarization states)

∴ Eleventh commandment: Thou Shallst Not Mess With Thyne or Thyne Neighbors Gauge Invariance!

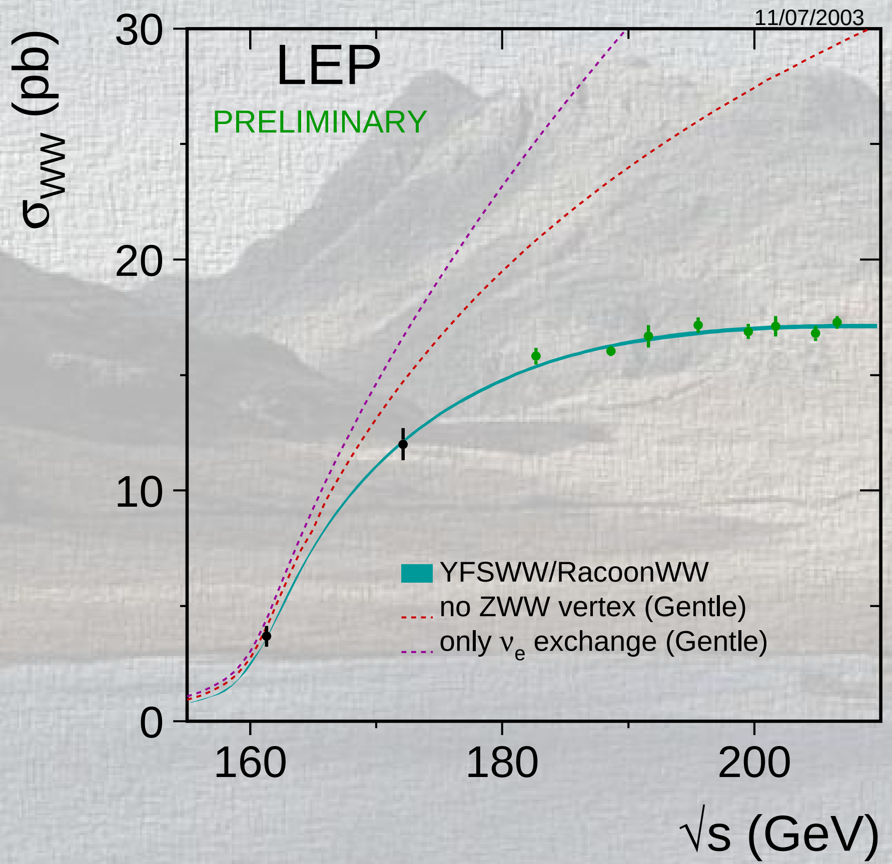
Remember LEP2:



- only the sum of all diagrams in $e^+e^- \to W^+W^-$ gives a correct result



- leaving out any one diagram destroys the subtle destructive interferences and gives a completely wrong result:



11/07/2003
LEP
PRELIMINARY
σ_{WW} (pb)
30
20
10
0
160
180
200
√s (GeV)
YFSWW/RacoonWW
no ZWW vertex (Gentle)
only ν_e exchange (Gentle)

Remember Stefan's lecture, where he deduced the Electro Weak Standard Model from observations.

☺ "facts"

- low energy experiments have established the gauge symmetry structure as a spontaneous breaking $\mathrm{SU}_L(2) \otimes \mathrm{U}_Y(1) \to \mathrm{U}_Q(1)$ based on the observations of
  - (partially) conserved $\mathrm{SU}_L(2)$ currents with vector exchange
  - short range interaction
  - Fermi scale $v = 246\,\mathrm{GeV}$

spontaneously broken gauge symmetry

- only (known) physically and mathematically consistent description of interacting vector bosons: cancellation of unphysical degrees of freedom

☹ fiction

- linear realization of EWSB
- multiplet(s) of elementary scalar particles with

$$V(\Phi) = \frac{1}{4}\left(\Phi^2 - v^2\right)^2$$

- unpleasant features:
  - ∗ not natural unless stabilized by SUSY

☹☹☹ SUSY efficiently camouflages flavor physics

∴ Planck Scale Collider required for flavor physics (wherever the Planck scale is . . .)

☺ no-loose prediction for interesting TeV-scale EWSB physics:

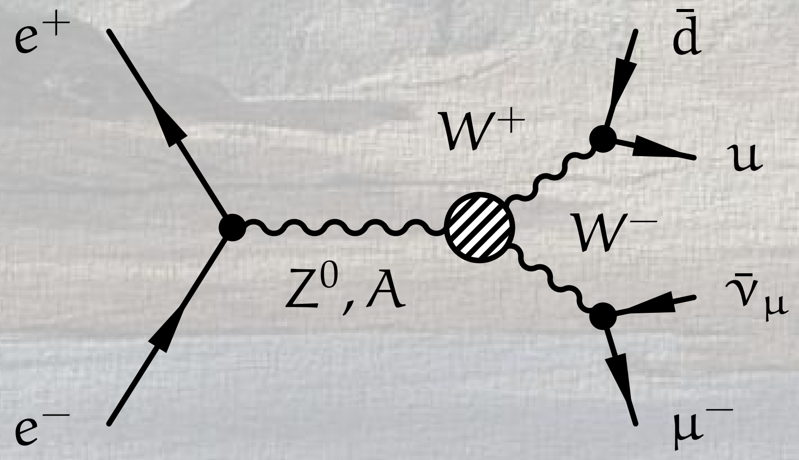
- if there is no light Higgs-like resonance, unitarity requires strong electroweak interactions at

$$\Lambda_{\text{EWSB}} = 4\pi v \approx 3\,\text{TeV}$$

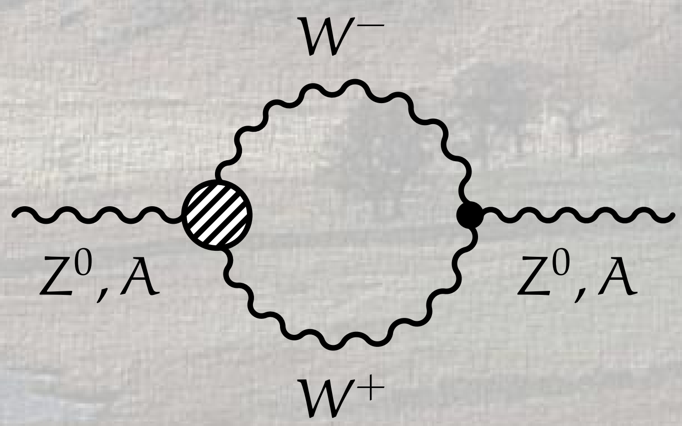
popular pastime of the 1990ies:

- parametrize the most general gauge boson vertices and try to measure the associated form factors

tree level scattering amplitudes:



and (sometimes) even loops:



☹ arbitrary form factor shapes obstruct all identifications of physical scales

☹ explicit symmetry breaking mixes up hierarchies

Effective Field Theory provides a more systematic approach:

- write all interactions as local interactions
  - ☺ well defined perturbation theory
  - ☺ well defined renormalization prescription in each order of external energies/momenta
  - ☺ only a finite number of parameters in each order of external energies/momenta
- classify local operators according to their naive dimension

$$\mathcal{L} = \ldots + \frac{g_i^{(n)}}{\Lambda_{\text{SB}}^n} \mathcal{O}_i^{(n)} + \ldots$$

- Naive Dimensional Analysis (NDA) shows that Λ_{SB} is the symmetry breaking scale, if naturally $g_i^{(n)} = O(1)$

☺ all unnatural small parameters have been replaced by hierarchies of (physical) symmetry breaking scales

How does it work?

- fields, derivatives and lagrangians have well defined (perturbative) dimensions

$$\text{dim}(\phi) = \text{dim}(A_\mu) = \text{dim}(\partial_\mu) = 1$$
$$\text{dim}(\psi) = \text{dim}(\bar\psi) = \frac{3}{2}$$
$$\text{dim}(\mathcal{L}) = 4$$

- familiar examples of operators:

$$\text{boson/fermion mass terms}: \quad \frac{1}{2}m^2\phi^2,\ m\bar\psi\psi$$
$$\text{boson/fermion kinetic terms}: \quad \frac{1}{2}(\partial\phi)^2,\ \bar\psi i\partial\!\!\!/\psi$$
$$\text{Higgs self coupling}: \quad \lambda\phi\phi\phi$$
$$\text{Yukawa/gauge couplings}: \quad g\phi\bar\psi\psi,\ g\phi\bar\psi A\!\!\!/\psi$$
$$\text{Fermi coupling}: \quad G_F\bar\psi\gamma_\mu(1-\gamma_5)\psi\bar\psi\gamma_\mu(1-\gamma_5)\psi$$

- general case:

$$\delta\mathcal{L} = 16\pi^2 v^4 \left(\frac{A_\mu}{4\pi v}\right)^{n_A} \left(\frac{\partial_\mu}{4\pi v}\right)^{n_\partial} \left(\frac{\phi}{v}\right)^{n_\phi} \left(\frac{\psi}{\sqrt{4\pi v}v}\right)^{n_\psi} \left(\frac{\bar\psi}{\sqrt{4\pi v}v}\right)^{n_{\bar\psi}}$$

(where the counting of the 4π involves a bit of—legitimate—magic)

- two points of view

  - operators with dimension > 4 (i. e. with coupling constants of negative dimensions), like the Fermi coupling are evil, because they require counter terms of even higher dimension to renormalize the theory

  - matrix elements of operators with dimension > 4 are suppressed by powers of

$$\left(\frac{E}{4\pi v}\right)$$

    and can be controlled since the counter term contributions will be suppressed even stronger.

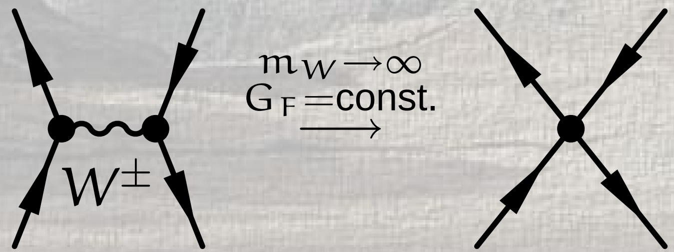
- both points of view are correct, in their respective region of applicability.

- demanding renormalizability is the correct approach, when
  - there is no higher energy scale that is important
  - renormalizability means that all dependence on physics at a higher scale can be absorbed into a finite number of counter terms
    - masses
    - wave function renormalizations
    - gauge couplings
    - Yukawa couplings
    - triple scalar self couplings
    - quartic scalar self couplings

- the effective field theory approach is correct, when
  - we study the physics much below an important higher energy scale $4\pi v$
  - the higher dimension operators are irrelevant and can be studied by an expansion in $(E/4\pi v)$
  - the physics at the higher scale can be relied upon to provide all necessary counter terms
  - the relevant operators (like masses) are troublesome, because their matrix elements blow up like $(4\pi v/E)$ at low

energies (naturalness problem).

- top to bottom approach
  - effective theory derived from a known more fundamental theory
  - classic example: Fermi interaction



☺ calculations often simpler in the effective theory

$$\frac{i}{p^2 - m_W^2 + i\epsilon} \to -\frac{i}{m_W^2} + \mathcal{O}\left(\frac{p^2}{m_W^2}\right)$$

∴ powerful tool in hadronic physics

- bottom to top approach
  - more fundamental theory is unknown

∴ just use all operators with the correct symmetry at the given order in the energy expansion

☺ all required counter terms will be there (by the magic of power counting)

☹ less predictive power than a fundamental theory

☺ still non trivial predictions, since there is only a finite number of couplings in each order of the energy expansion

$$\mathcal{L} = v^2 \operatorname{tr}\left((\partial_\mu U)^\dagger \partial^\mu U\right), \quad \text{with } U(x) = e^{i\phi(x)/v}$$

therefore

$$\mathcal{L} = \operatorname{tr}\left((\partial_\mu \phi)^\dagger \partial^\mu \phi\right) + \frac{1}{6v^2} \operatorname{tr}\left([\phi, \partial_\mu \phi]^\dagger [\phi, \partial^\mu \phi]\right) + \cdots$$

- $\mathcal{L}$ contains vertices of arbitrarily high order
- it is impossible to write a mass term (NB: $\operatorname{tr}(U^\dagger U) = \text{const.}$)

Example $SU(2)_L \otimes SU(2)_R$: $U(x)$ shall transform according to $SU(2)_L$ from the left and $SU(2)_R$ from the right

$$U(x) \to L U(x) R^\dagger$$

then ϕ transforms linearly for $L = R$

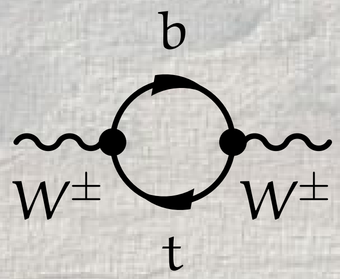
$$\phi(x) \to L\phi(x)L^\dagger = \phi + [\ln L, \phi(x)] + \cdots$$

but nonlinearly for $L = R^\dagger \neq R$, i. e. $\phi(x) \to \phi + \ln L + \cdots$

- $\phi(x)$ is just like a Goldstone boson and can give mass to gauge bosons, but without a Higgs

Experimental observation:

- $\rho = m_W^2/(m_Z \cos\theta_w)^2 \approx 1$
- with today's precision, $\Delta\rho = \rho - 1$ can be accounted for entirely by electroweak loop effects



$\propto (m_t^2 - m_b^2)$

∴ the small $\Delta\rho$ can not be accidental

☺ $\rho \approx 1$ can be explained by a custodial $SU(2)_c$ symmetry under which (W^1, W^2, W^3) transform like a vector

☹ *the uncensored director's cut:* $SU(2)_c$ is explicitely broken by the electric charge, but we'll ignore this for a moment

∴ electroweak symmetry breaking sector

$$SU(2)_L \otimes SU(2)_R \to SU(2)_c$$

- economical implementation of partially broken custodial symmetry using the exponential representation of the Goldstone boson fields and $SU(2)_L \times U(1)_Y$ covariant derivatives

$$U(x) = e^{-i\sigma^k \phi^k(x)/v}, \quad D_\mu = \partial_\mu - ig\frac{\sigma^i}{2}W_\mu^i + ig'\frac{Y}{2}B_\mu, \quad V_\mu(x) = U^\dagger(x) D_\mu U(x)$$

and field strengths as building blocks

- $L \in SU(2)_L \times U(1)_Y$ and $R \in SU(2)_c$: $U \to LUR$

  $\therefore\ V \to V$ (under $SU(2)_L \times U(1)_Y$)

  $\therefore\ V \to R^\dagger V R$ (under $SU(2)_c$)

- all $SU(2)_c$-invariant interactions of vector bosons can be constructed systematically as products of traces of products of V_μs and the field strengths.

- NB: in unitarity gauge

$$V_\mu = -ig\frac{\sigma^i}{2}W_\mu^i + ig'\frac{\sigma^3}{2}B_\mu$$

- Three operators contain no σ_3s and are explicitly $SU(2)_c$ conserving
- Two operators break $SU(2)_c$ linearly with one explicit σ_3s
- Four operators break $SU(2)_c$ quadratically with two explicit σ_3s
- One operator breaks $SU(2)_c$ quartically with four explicit σ_3s

$$L_1 = \frac{\alpha_1}{16\pi^2}\frac{gg'}{2}B_{\mu\nu}\,\mathrm{tr}\left(\sigma_3 W^{\mu\nu}\right) \qquad L_6 = \frac{\alpha_6}{16\pi^2}\,\mathrm{tr}\left(V_\mu V_\nu\right)\mathrm{tr}\left(\sigma_3 V^\mu\right)\mathrm{tr}\left(\sigma_3 V^\nu\right)$$

$$L_2 = \frac{\alpha_2}{16\pi^2}ig'B_{\mu\nu}\,\mathrm{tr}\left(\sigma_3 V^\mu V^\nu\right) \qquad L_7 = \frac{\alpha_7}{16\pi^2}\,\mathrm{tr}\left(V_\mu V^\mu\right)\mathrm{tr}\left(\sigma_3 V_\nu\right)\mathrm{tr}\left(\sigma_3 V^\nu\right)$$

$$L_3 = \frac{\alpha_3}{16\pi^2}2ig\,\mathrm{tr}\left(W_{\mu\nu}V^\mu V^\nu\right) \qquad L_8 = \frac{\alpha_8}{16\pi^2}\frac{g^2}{4}\,\mathrm{tr}\left(\sigma_3 W_{\mu\nu}\right)\mathrm{tr}\left(\sigma_3 W^{\mu\nu}\right)$$

$$L_4 = \frac{\alpha_4}{16\pi^2}\,\mathrm{tr}\left(V_\mu V_\nu\right)\mathrm{tr}\left(V^\mu V^\nu\right) \qquad L_9 = \frac{\alpha_9}{16\pi^2}\frac{ig}{2}\,\mathrm{tr}\left(\sigma_3 W_{\mu\nu}\right)\mathrm{tr}\left(\sigma_3 V^\mu V^\nu\right)$$

$$L_5 = \frac{\alpha_5}{16\pi^2}\,\mathrm{tr}\left(V_\mu V^\mu\right)\mathrm{tr}\left(V_\nu V^\nu\right) \qquad L_{10} = \frac{\alpha_{10}}{16\pi^2}\,\mathrm{tr}\left(\sigma_3 V_\mu\right)\mathrm{tr}\left(\sigma_3 V_\nu\right)\mathrm{tr}\left(\sigma_3 V^\mu\right)\mathrm{tr}\left(\sigma_3 V^\nu\right)$$

$\because$ $SU(2)_c$ appears to be (approximately) conserved in EWSB

$\therefore$ any potential breaking must be governed by a higher scale $\Lambda_F > \Lambda_{EWSB}$, probably related to flavor physics *(if we're lucky...)*

- natural implementation
  - multiply each spurion (i. e. each explicit σ_3) by one power of Λ_{EWSB}/Λ_F.
- the coefficients α_i are related to scales of new physics in EWSB Λ_i^* by naive dimensional analysis (NDA)

$$\frac{\alpha_i}{16\pi^2} = \left(\frac{v}{\Lambda_i^*}\right)^2$$

- in the absence of resonances that are lighter than $4\pi v$, one expects from NDA in a strongly interacting symmetry breaking sector

$$\Lambda_i^* \approx \Lambda_{EWSB} = 4\pi v \approx 3\,\mathrm{TeV}\,, \quad \text{i.e.} \quad \alpha_i \approx \mathcal{O}(1)\,,$$

$\therefore$ crucial benchmark

$$g_1^Z = 1 + \frac{e^2}{\cos^2\theta_w(\cos^2\theta_w - \sin^2\theta_w)}\frac{\alpha_1}{16\pi^2} + \frac{e^2}{\sin^2\theta_w\cos^2\theta_w}\frac{\alpha_3}{16\pi^2}$$

$$\kappa_Z = 1 + \frac{2e^2}{\cos^2\theta_w - \sin^2\theta_w}\frac{\alpha_1}{16\pi^2} - \frac{e^2}{\cos^2\theta_w}\frac{\alpha_2}{16\pi^2} + \frac{e^2}{\sin^2\theta_w}\frac{\alpha_3}{16\pi^2}$$

$$\kappa_\gamma = 1 - \frac{e^2}{\sin^2\theta_w}\frac{\alpha_1}{16\pi^2} + \frac{e^2}{\sin^2\theta_w}\frac{\alpha_2}{16\pi^2} + \frac{e^2}{\sin^2\theta_w}\frac{\alpha_3}{16\pi^2}\,.$$

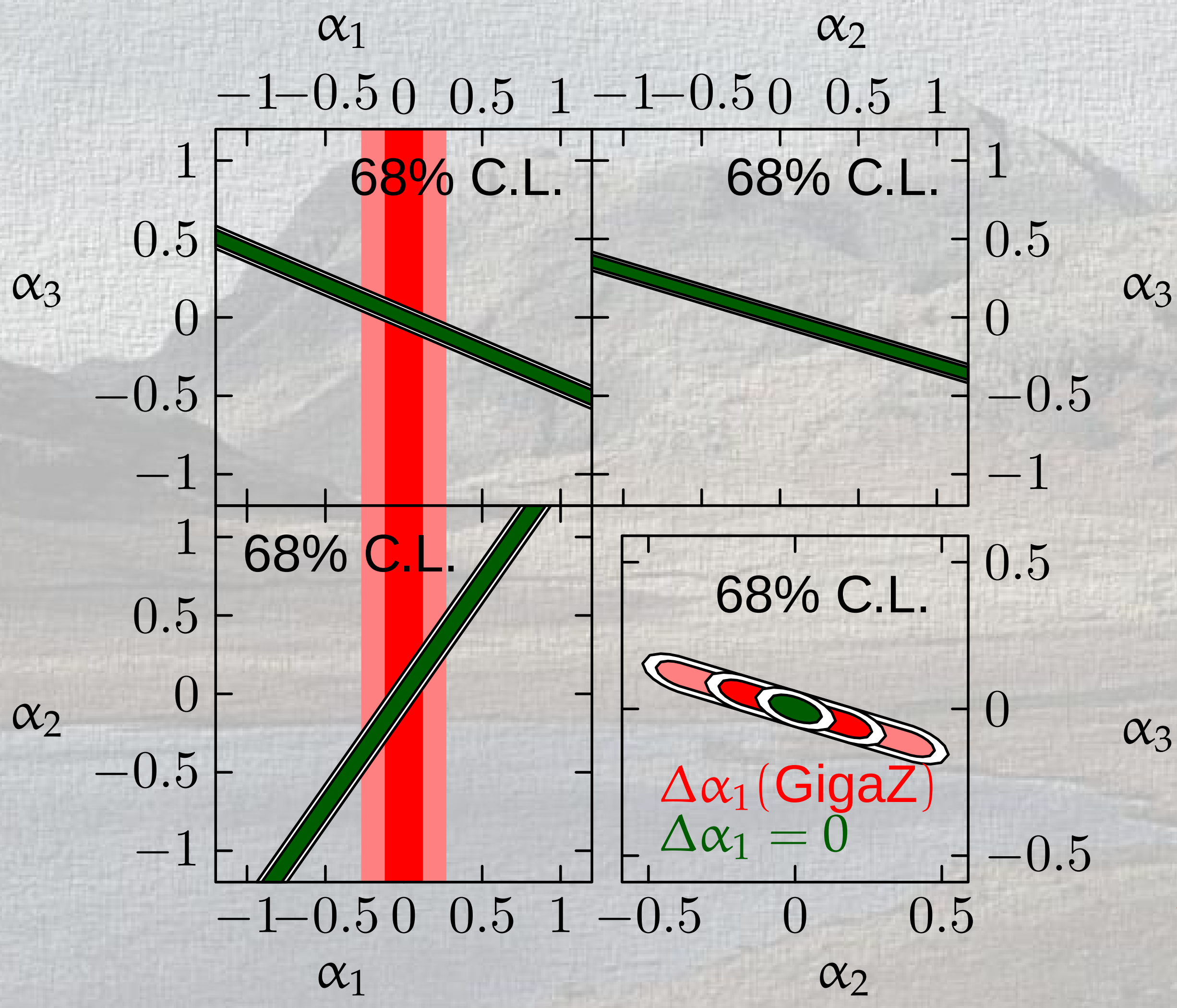
- This transformation is singular and the resulting anomalous couplings satisfy

$$(\Delta g_Z^1 - \Delta\kappa_Z)\cdot\cos^2\theta = \Delta\kappa_\gamma\cdot\sin^2\theta.$$

Only two dimensions of the $\alpha_{1,2,3}$ parameter space can be determined directly in four-fermion production. The blind direction

$$(\alpha_1,\ \alpha_2,\ \alpha_3)_{\text{blind}} \propto (\cos^2\theta_w - \sin^2\theta_w,\ \cos^2\theta_w,\ -\sin^2\theta_w)$$

in the parameter space can not be constrained from TGCs alone.



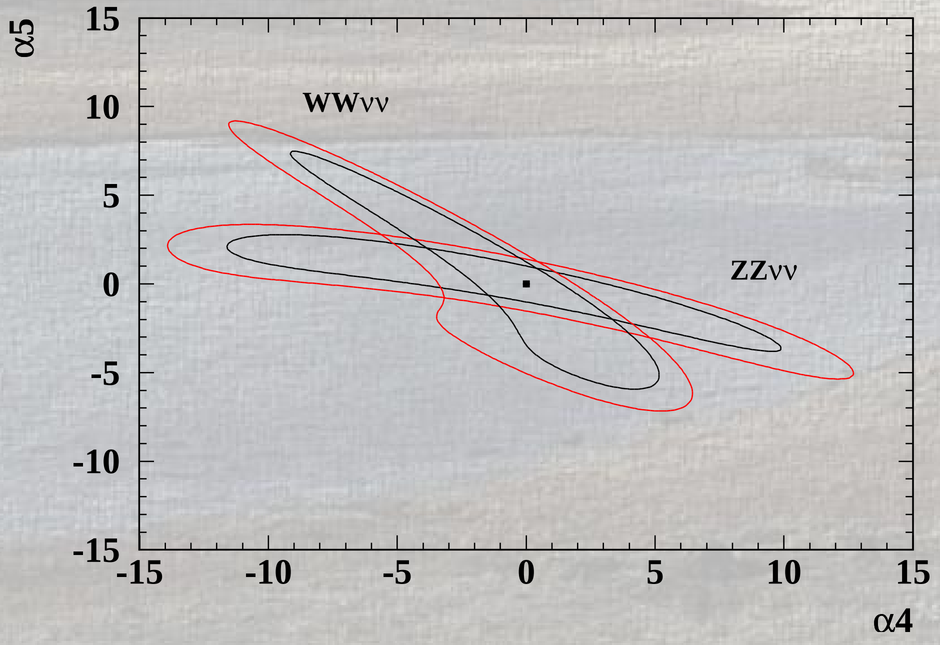
Sensitivity for strong EWSB parameters at TESLA with $800\,\mathrm{GeV}$, $1000\,\mathrm{fb}^{-1}$, $\mathrm{P}_{e^-} = 80\%$, and $\mathrm{P}_{e^+} = 60\%$. The inner shaded diagonals correspond to $\Delta\chi^2 = 1$ and the outer diagonals correspond to 68% C.L.. The dark and light vertical bands are the 68% C.L. limits on α_1 from fitting ε_3 at GigaZ with and without the constraint $\varepsilon_2 = \varepsilon_2(\mathrm{SM})$.

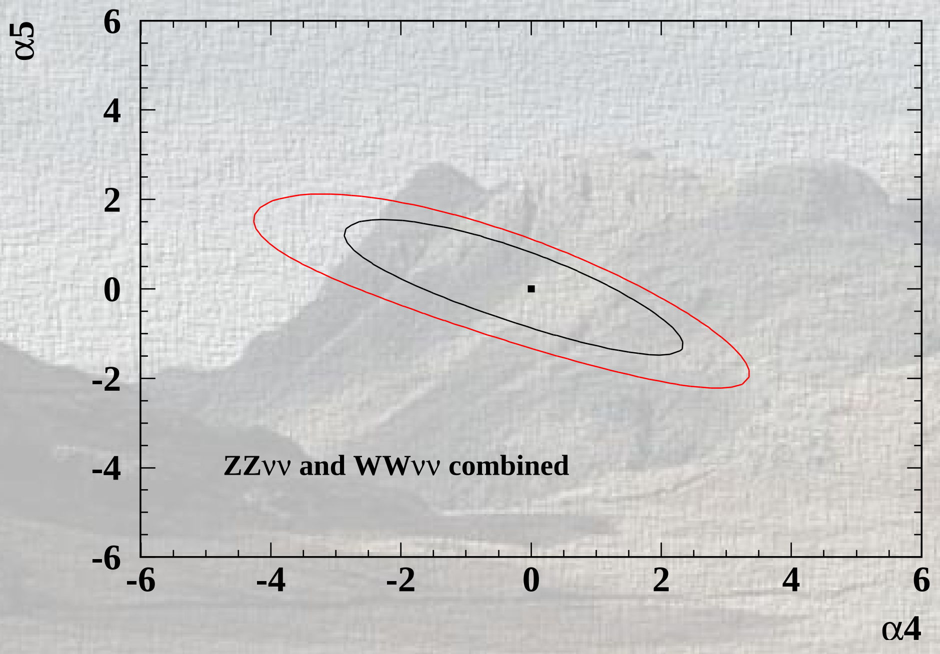
$\alpha_1 = 0$	$P_{e^-} = 80\%, P_{e^+} = 0\%$		$P_{e^-} = 80\%, P_{e^+} = 60\%$	
$\sqrt{s}$	500 GeV	800 GeV	500 GeV	800 GeV
$\int \mathcal{L} dt$	500 fb^{-1}	1000 fb^{-1}	500 fb^{-1}	1000 fb^{-1}
$\Delta\alpha_2$	0.329	0.127	0.123	0.090
$\Delta\alpha_3$	0.143	0.071	0.083	0.048
Λ_2^*	5.4 TeV	8.7 TeV	8.8 TeV	10.3 TeV
Λ_3^*	8.2 TeV	11.6 TeV	10.7 TeV	14.1 TeV

68% C.L. sensitivities for the strong EWSB parameters (α_2, α_3), assuming $\alpha_1 = 0$, in a study of TGCs at a TESLA experiment, with and without positron polarization.

$$\mathcal{L}_4 = \frac{\alpha_4}{16\pi^2}\left(\frac{g^4}{2}\left((W^+_\mu W^{-,\mu})^2 + W^+_\mu W^{+,\mu} W^-_\mu W^{-,\mu}\right) + \frac{g^4}{\cos^2\theta_w} W^+_\mu Z^\mu W^-_\nu Z^\nu + \frac{g^4}{4\cos^4\theta_w}(Z_\mu Z^\mu)^2\right)$$

$$\mathcal{L}_5 = \frac{\alpha_5}{16\pi^2}\left(g^4 (W^+_\mu W^{-,\mu})^2 + \frac{g^4}{\cos^2\theta_w} W^+_\mu W^{-,\mu} Z_\nu Z^\nu + \frac{g^4}{4\cos^4\theta_w}(Z_\mu Z^\mu)^2\right)$$




α5
6
4
2
0
-2
-4
-6
ZZνν and WWνν combined
-6
-4
-2
0
2
4
6
α4

68% C.L. sensitivities from one dimensional fits of the strong EWSB parameters (α_4, α_5) at LHC and TESLA:

$\sqrt{s}$	LHC	TESLA 800 GeV
$\int \mathcal{L} dt$	$100\,\mathrm{fb}^{-1}$	$1000\,\mathrm{fb}^{-1}, P_{e^-} = 80\%, P_{e^+} = 40\%$
α_4	$-0.17 \ldots +1.7$	$-1.1 \ldots +0.8$
α_5	$-0.35 \ldots +1.2$	$-0.4 \ldots +0.3$
Λ_4^*	2.3 TeV	2.9 TeV
Λ_5^*	2.8 TeV	4.9 TeV

The TESLA result is based on a full tree level calculation and fast simulation.

Are there some even more exotic proposals?

☺ glad you asked . . .

E. g. non-commutative space-time $x_\mu \to \hat{x}_\mu$:

$$[\hat{x}_\mu, \hat{x}_\nu] = \frac{i\theta_{\mu\nu}}{\Lambda^2}$$

with

- $\theta_{\mu\nu} = -\theta_{\nu\mu}$
- $\Lambda = \mathcal{O}(1\,\mathrm{TeV}) \ldots 10^{19}\,\mathrm{GeV}$

(for all practical purposes) this looks like an associative Moyal $*$-product

$$f(x)g(x) \to (f*g)(x) = \lim_{\xi,\eta\to 0} \left[e^{i\partial_\xi \wedge \partial_\eta} f(x+\xi)g(x+\eta)\right]$$

and decorates the vertices with momentum dependent phase factors $e^{ip\wedge q}$ using

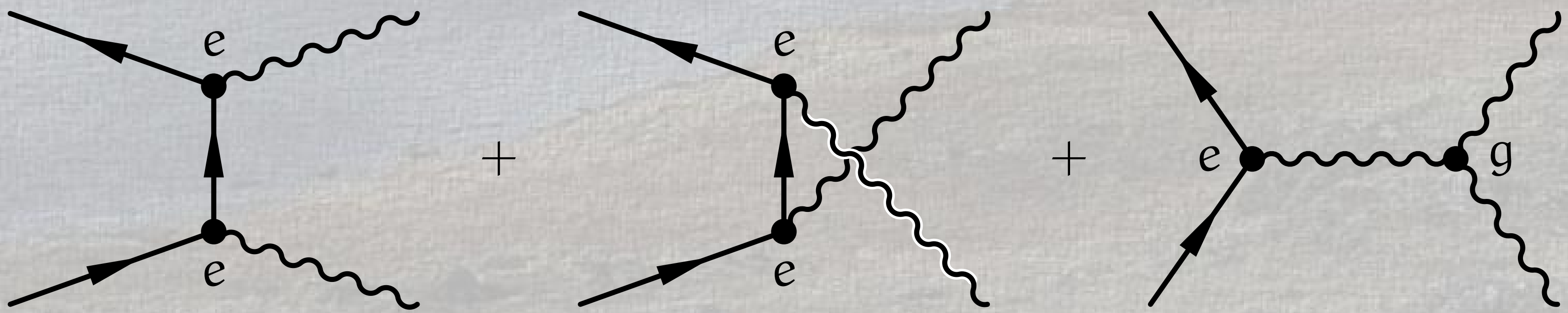
$$p \wedge q = \frac{1}{2\Lambda^2} p^\mu \theta_{\mu\nu} q^\nu \,.$$

Why is this interesting?

- “because we can” (Steven Weinberg)
- consistent field theory with a fundamental length (Wess)
- low energy limit of string theory (Seiberg/Witten)
  - think of dipoles as lowest order approximations to open strings
  - it matters which ends interact

Consequences:

- self couplings of neutral gauge bosons



"Not so fast" (Wess et al.)

- more careful construction relaxes the condition $g = e$ and gives an allowed range instead.

☺ model remains predictive

Observation:

☹ nothing has been seen so far

☺ current limits are still rather soft: $\Lambda = \mathcal{O}(1\,\mathrm{TeV})$

☺ Linear Collider is the ideal tool for finding the neutral gauge boson couplings

☹ if $\Lambda \gg \mathcal{O}(10\,\mathrm{TeV})$ we will have no chance with terrestrial accelerators